AS-2767 B.Sc. I^{st} Sem. Paper: First

Physics Mechanics & Properties of Matter

Section A

①

Ans ① (i) b away from the centre

(ii) d All the above

(iii) d interaction of e.m. waves with ionosphere

(iv) a inelastic

(v) a $Kg\text{-}m^2$

(vi) a spherical top

(vii) b within elastic limit, stress ∝ strain

(viii) c $ML^{-1}T^{-2}$

(ix) c Torsional rigidity

(x) a decreases

Section B.

Ans (2) The work done by the force in moving the particle for a displacement $d\vec{s}$ is (at any time t)

$$dW = \vec{F}\cdot d\vec{s}$$

$$= \vec{F}\cdot\frac{d\vec{s}}{dt}\,dt = \vec{F}\cdot\vec{v}\,dt$$

∴ work done by the force in moving the particle when force is acted from time $t=0$ to $t=t$ sec is

$$W = \int_0^t dW = \int_0^t \vec{F}\cdot\vec{v}\,dt$$

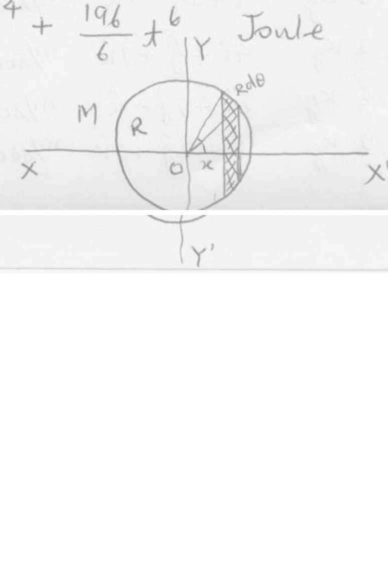
$$= \int_0^t (9t^3\hat{i}+3t\hat{j}-7t^2\hat{k})\cdot(27t^4\hat{i}+18t^2\hat{j}-28t^3\hat{k})\,dt$$

$$= \int_0^t (243t^7+54t^3+196t^5)\,dt$$

$$= \left(\frac{243}{8}t^8+\frac{54}{4}t^4+\frac{196}{6}t^6\right)_0^t$$

$$= \frac{243}{8}t^8+\frac{54}{4}t^4+\frac{196}{6}t^6 \text{ Joule}$$

Ans (3) Consider a thin circular ring of thickness dx at a distance x from O.



Radius of ring $= \sqrt{R^2-x^2}$

mass per unit area of spherical shell $= \frac{M}{4\pi R^2}$ ②

mass of the thin circular ring

$$= \frac{M}{4\pi R^2} \, 2\pi R \sin\theta \cdot R\, d\theta$$

put $x = R\cos\theta$

$\therefore \quad dx = R\sin\theta \, d\theta$ (As x increases θ decreases)

$\therefore$ mass of the thin circular ring $= \frac{M}{2R} dx$

$\therefore$ Moment of inertia of thin circular ring about xx' is

$$= \frac{M}{2R} dx \left(R^2 - x^2\right)$$

$\therefore$ Moment of inertia of spherical shell about xx' is

$$= \int_{-R}^{R} \frac{M}{2R}\left(R^2 - x^2\right) dx = \frac{M}{2R}\left(R^2 x - \frac{x^3}{3}\right)_{-R}^{R}$$

$$= \frac{2}{3} M R^2$$

Moment of inertia of the spherical shell about the tangent

$$= \frac{2}{3} MR^2 + MR^2 = \frac{5}{3} MR^2$$

Ans (4) velocity of centre of mass of a system of N particles of masses $m_1, m_2, \cdots\cdots m_N$ whose velocities are $\vec{v}_1, \vec{v}_2, \cdots\cdots \vec{v}_N$ is defined as

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots\cdots + m_N\vec{v}_N}{m_1 + m_2 + \cdots + m_N} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{v}_i$$

mass	velocity
1 Kg	$4\hat{i} + 2\hat{j} - 7\hat{k}$ m/sec
3 Kg	$\hat{i} + 2\hat{j} - 7\hat{k}$ m/sec
2 Kg	$4\hat{i} + \hat{j} + 7\hat{k}$ m/sec
6 Kg	$2\hat{i} + 9\hat{j} - \hat{k}$ m/sec
9 Kg	$\hat{i} + 2\hat{j} + 3\hat{k}$ m/sec

$$\vec{V}_{cm} = \frac{36\hat{i} + 82\hat{j} + 7\hat{k}}{21} \text{ m/sec}$$

Ans(5) Elasticity is the property of matter by virtue of which a body regains the original shape or size on the removal of deformation forces.

If the body recovers completely its original condition as soon as the deforming forces have been removed, it is said to be perfectly elastic while if it completely retains its modified form, it is said to be perfectly plastic. There are however, no perfectly elastic or plastic bodies and actual bodies lie between the two extremes.

The bodies are said to be elastic as long as they follow the Hook's law. According to this law within the elastic limit, stress is proportional to strain

$\Rightarrow$ stress $\propto$ strain

$$\Rightarrow \underbrace{\frac{\text{stress}}{\text{strain}} = \text{Constant}}_{\text{Modulus of elasticity}}$$

There are three types of elastic modulie.

(i) Young's modulus $(Y) = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}} = \dfrac{Fl}{A\,\delta l}$

(ii) Bulk modulus $(K) = \dfrac{\text{Normal stress}}{\text{Volume strain}} = -\dfrac{FV}{A\,\delta v}$

(iii) Modulus of rigidity $(\eta) = \dfrac{\text{Tangential stress}}{\text{shearing strain}} = \dfrac{F}{A\theta} = \dfrac{Fl}{A\,\delta l}$

Now take two wires of copper and rubber having same length and area of cross-section. Now apply equal force on the both wires

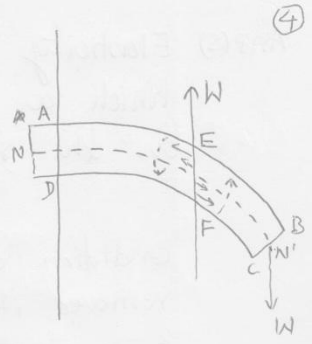
$$Y_{copper} = \frac{Fl}{A(\delta l)_c}\ ,\quad Y_{rubber} = \frac{Fl}{A(\delta l)_r}$$

Here $(\delta l)_{copper} < (\delta l)_{rubber}$ $\because$ for copper the change in the length is smaller.

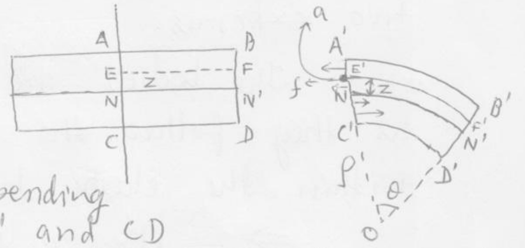
$\Rightarrow$ Y_{copper} is greater than Y_{rubber}

$\Rightarrow$ Copper is more elastic than rubber.

Ans 6 When a beam is bent by an external applied couple, an internal couple (restoring couple) is developed at each cross-section of the beam due to its elasticity. In the equilibrium state, the restoring couple is equal and opposite to the external couple.



The moment of the restoring couple is called as the bending moment of the beam. It is equal to moment of the bending couple.



Consider a small portion of a beam bounded by two sections AC and BD. After bending AB is elongated to A'B' and CD is compressed to C'D'. NN' is the Neutral surface. Consider a filament EF at a distance z from NN'. a be the area of cross-section of the filament and f be the force acting on the cross sectional area of the filament. EF is elongated to E'F'. The bent beam A'B'C'D' subtend an angle θ at the centre of curvature O. ρ be the radius of curvature. Then

$$\text{longitudinal stress} = \frac{f}{a}$$

$$\text{change in length of filament} = E'F' - EF = (\rho+z)\theta - \rho\theta$$
$$= z\theta$$

$$\text{longitudinal strain} = \frac{z\theta}{\rho\theta} = \frac{z}{\rho}$$

$$\therefore \text{Young's modulus } Y = \frac{f/a}{z/\rho}$$

$$\Rightarrow f = \frac{Ya}{\rho} z$$

Moment of this force about neutral surface $= fz$

$$= \frac{Ya}{\rho} z^2$$

The sum of moments of the forces acting over the whole section A'C' is the magnitude of the bending moment

$$\therefore \quad \text{bending moment} = \sum fz = \sum \frac{Ya}{\rho} z^2$$

$$= \frac{Y}{\rho} \sum az^2$$

$$= \frac{Y}{\rho} I$$

where $I = \sum az^2$ is called as the geometrical moment of inertia of the section about the neutral surface of the beam.

Ans (7) Stoke's Law

when a small sphere of radius r is moving slowly with a constant velocity v through a homogeneous viscous fluid of infinite extension a viscous force is acting on the sphere. Stoke derived a formula for the viscous force acting on the sphere by dimensional equation. According to Stoke, the viscous force acting on the sphere is given as $\boxed{F = 6\pi \eta r v}$ where η is the coefficient of viscosity of fluid, r is the radius of ~~ball~~ sphere, v is the constant velocity of the sphere in the fluid.

According to stoke:

$$\text{Viscous Force} \not\propto K r^a \eta^b v^c$$

where K is constant

Writing the dimension on both sides.

$$[MLT^{-2}] = L^a [ML^{-1}T^{-1}]^b [LT^{-1}]^c$$

or, $MLT^{-2} = M^{a} L^{a-b+c} T^{-b-c}$

Equating the powers on both sides, we get

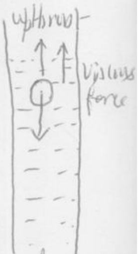
$$b = 1$$
$$a-b+c = 1 \quad \Rightarrow \quad a=1,\ b=1,\ c=1$$
$$-b-c = -2$$

If the extension of fluid is infinite, then

$$K = 6\pi$$

$$\therefore \quad \boxed{F = 6\pi\eta r v}$$

If the density of material of ball is ρ,
the weight of the ball $= \frac{4}{3}\pi r^3 \rho g$



The upthrust on the ball due to displacement of the fluid is $= \frac{4}{3}\pi r^3 \sigma g$
where σ is the density of fluid.

At terminal velocity $v = v_T$ (constant velocity)

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 (\rho - \sigma) g$$

$$\therefore \quad \boxed{v_T = \frac{2}{9} r^2 g \frac{(\rho-\sigma)}{\eta}}$$

Ans (8) Given $A_1 = 0.02\ m^2$, $A_2 = 0.01\ m^2$
$v_1 = 2\ m/sec$ $\quad v_2 = ?$

Equation of continuity $A_1 v_1 = A_2 v_2$

$$\therefore \quad 2 \times 0.02 = 0.01 \times v_2$$

$$\Rightarrow \quad v_2 = 4\ m/sec$$